Mastery Based Assessment in Mathematics

Jason Elsinger, Ph.D.

Department of Mathematics
Florida Southern College

A Critique of Averages

Do these averages properly describe the abilities of the student?

	Exam 1	Exam 2	Exam 3	Average
Student A	75	73	77	75
Student B	100	30	95	75
Student C	50	80	95	75
Student D	100	65	60	75

Averages lose information.

Standards Based Grading

The course is split into 2 types of *standards*:

Core and Supplementary

- Precalculus: Solve an inequality using a sign chart. (Core)
- DE: Solve a problem using Newton's law of cooling. (Supp.)

Key features:

- Assessments are graded by standard
- Assessments use a binary rubric
- Students can initiate re-assessment opportunities
- Re-assessments can replace earlier scores

Marking Attempts

Each standard attempted is graded using an EMRN rubric:

- No Evidence: requires major revision/incomplete
- Re-assessment Needed: demonstrates some understanding but revisions are needed
- Meets Expectation: completely correct
- Exceeds Expectation (not always applicable)

In addition, we use a * mark for needs clarification/minor correction.

- alleviates frustration
- can be replaced with mastery

You try it!

Using the sheet provided, try writing a “big question” from one of your courses and some standards to go with it.

Grade Bundles in Differential Equations

Final Grade Checklist

Course grade	D	C	B	A
C standards mastered	☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐		☐ ☐ ☐ ☐
C standards continually mastered			☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐ ☐ ☐ ☐
S standards mastered		☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐ ☐	
Participation Score	☐50%	☐75%		
Homework Score	☐55%	☐75%	☐85%	☐95%

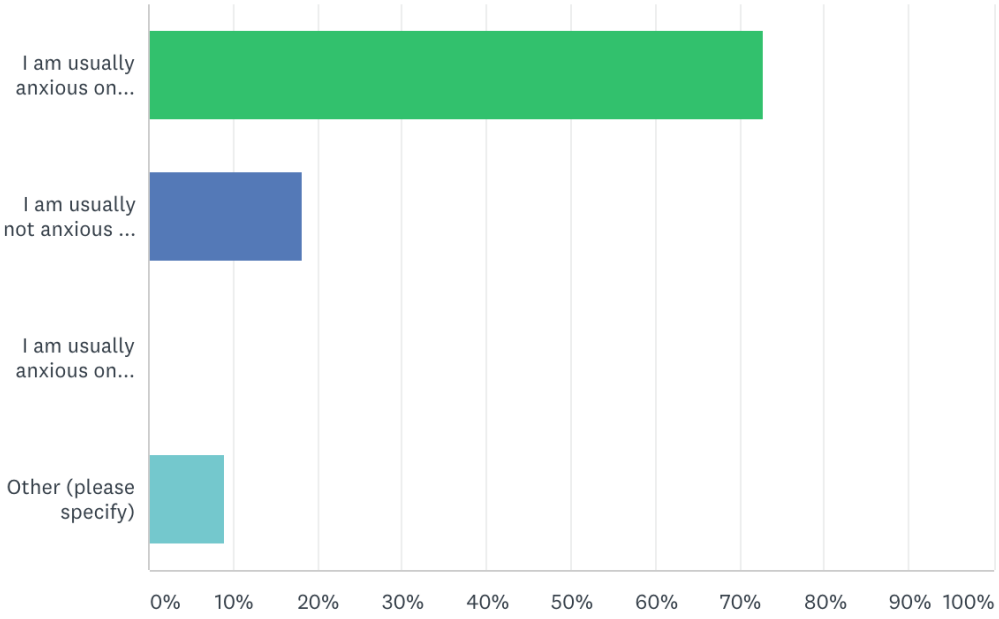
You try it!

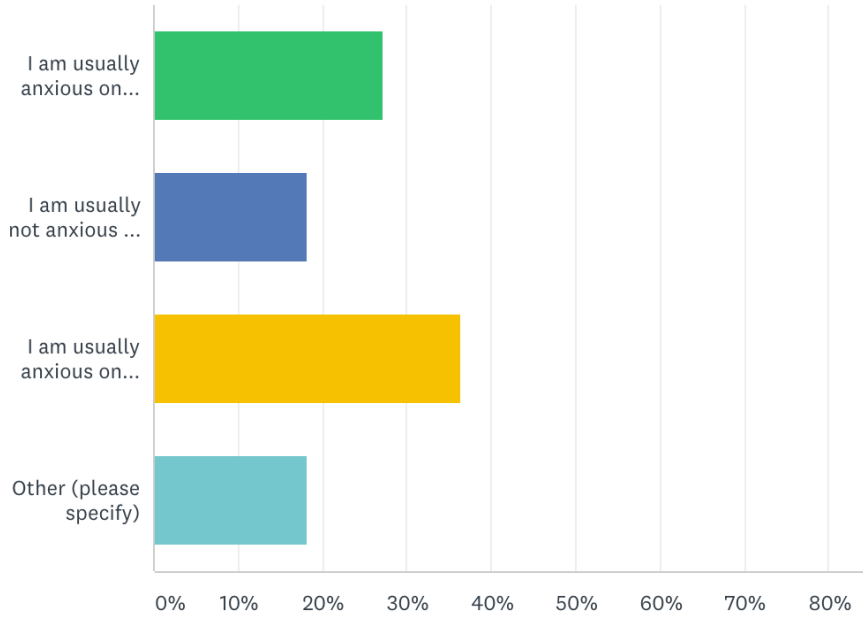
Using the sheet provided, invent content mastery and behavioral incentives you use for the same course.

Advantages

Some advantages of mastery-based exams:

- less stressful
- several chances to display mastery
- see which parts of the course deserve more attention

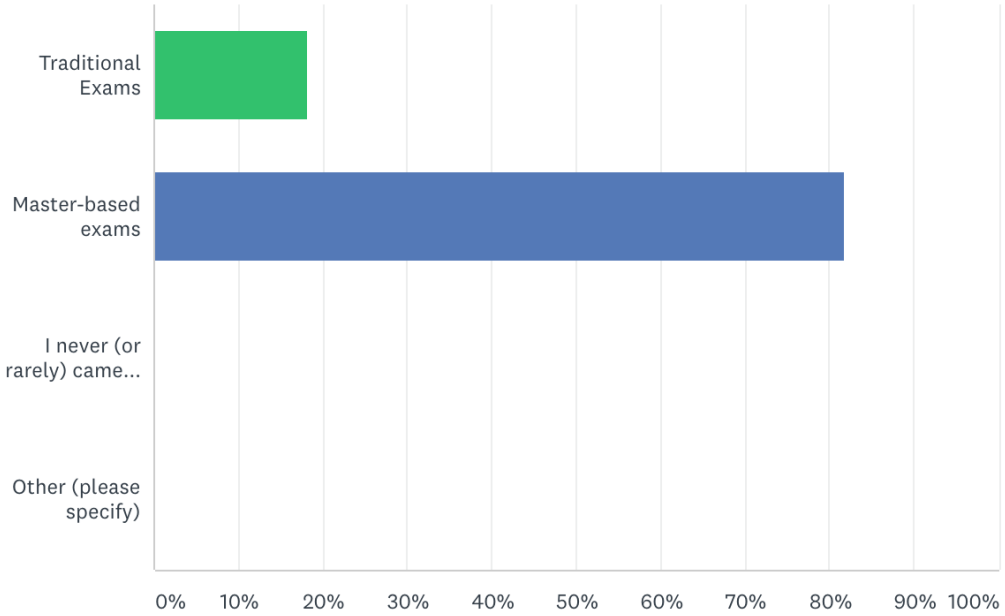


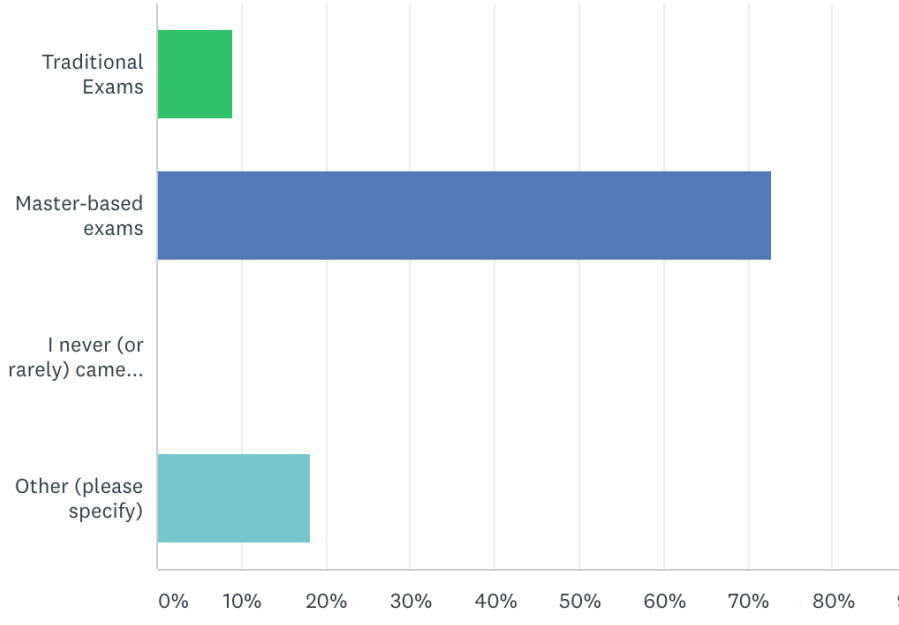


Advantages

Other advantages I observed:

- students learn from their mistakes
- became more aware of simple mistakes
- writing improved
- students came to office hours





Effects of Reassessment

Problem 2: solving inequalities

Attempt	Master	Journeyman	Apprentice
1	8	14	3
2	2	10	5
3	5	8	2
4	4	6	0

Problem 13: finding vertex, max/min, inc/dec

Attempt	Master	Journeyman	Apprentice
1	5	13	7
2	7	10	3
3	5	5	3
4	1	4	3

An Example from Differential Equations

S2: For a given initial value problem, determine all initial conditions for which the system will have a unique solution.

1. Determine the largest region in the xy-plane for which the differential equation would have a unique solution whose graph passess through the point (a, b).

$$\begin{cases} (\sqrt{y} - 1)y' = x \\ y(a) = b \end{cases}$$

$y^{\frac{1}{2}} = \frac{1}{2} y^{\frac{-3}{2}}$

$f_y(x) = \sqrt{y} - 1 \, dy$

$= \frac{1}{2} y^{\frac{3}{2}}$

the function x is cont. everywhere

$f_y(x) = (\sqrt{y} - 1) y'$

D

Attempts 2 & 3

S2: For a given initial value problem, determine all initial conditions for which the system will have a unique solution.

1. Determine the largest region in the xy-plane for which the differential equation would have a unique solution whose graph passess through the point (a, b).

$$\begin{cases} (x^2+y^2)y' = y^2 \\ y(a) = b \end{cases}$$

$$y' = \frac{y^2}{x^2+y^2} \leftarrow \text{cant} = 0$$

$$f_y = \frac{2y(x^2+y^2) - y^2(0+2y)}{(x^2+y^2)^2}$$

$$f_y = \frac{2y(x^2+y^2) - y^2(2y)}{(x^2+y^2)^2}$$

$$f_y = \frac{2yx^2+2y^3-2y^3}{(x^2+y^2)^2}$$

$$f_y = \frac{2yx^2}{(x^2+y^2)^2} \leftarrow \text{cant} = 0$$

$(x^2+y^2)(x^2+y^2)$

$y^4+x^2y^2+x^2y^2+y^4$

~~x and y are continuous everywhere when $x^2+y^2>0$~~

this is continuous when $x^2+y^2>0$

S2: For a given initial value problem, determine all initial conditions for which the system will have a unique solution.

1. Determine the largest region in the xy-plane for which the differential equation would have a unique solution whose graph passess through the point (a, b).

$$\begin{cases} (y-x)y' = y+x \\ y(a) = b \end{cases}$$

$$(y-x)y' = y+x$$

$$y' = \frac{y+x}{y-x} \rightarrow \text{cannot equal zero}$$

$$f_y = \frac{(1+0)(y-x) - (y+x)(1-0)}{(y-x)^2}$$

$$f_y = \frac{(y-x) - (y+x)}{(y-x)^2}$$

$$f_y = \frac{y-x-y-x}{(y-x)^2}$$

$$f_y = \frac{-2x}{(y-x)^2} \rightarrow \text{cannot equal zero}$$

∴ this is contin. when $y-x \neq 0$.

Hence by the theorem of IVP this has a unique solution for when $b-a \neq 0$.

Advice on Regulating Reassessments

Ways to reassess:

- in-class written assessment (quiz or exam)

  try frequent quizzes, less exams

- office hours written assessment

  Be organized! (and students too)

- office hours oral assessment

  limit to one attempt per day / more conversational

- reassessment form

Keeping a Gradebook

Sheet for attempts

Assessment	Q1	OH	OH	OH	Q2		OH		OH	Q3	OH						Exam 1								OH						Q4
Day Count	4	6	7		8	8		9		10	10	11					12								14						15
Standard	S1	S2	S1	S2	S2	C1	C2	C3	S1	S2	C3	C4	C1	C2	C3	C5	S2	C1	C2	C3	C4	C5	C6	S2	S4	C1	C4	C5	C6	S2	S4	C7
Student A	J			J		J	J					*/J						J		J		P	J									Y
Student B	J				P	J	D					*/J		J	P		*/J	J	J	J	P	*/J	P	P								P
Student C	Y	J				J	P				J	J		Y				J	*/J	J	J	P	P					J	*/J			P
Student D	D		J		D	D	D					D	P	J	*/J		P	P	*/J	D	P	P	D	P		J	J	J	P	J		D
Student F	J					J	P					J		Y	*/J		Y	*/J	J	J	*/J	P	J		J							Y

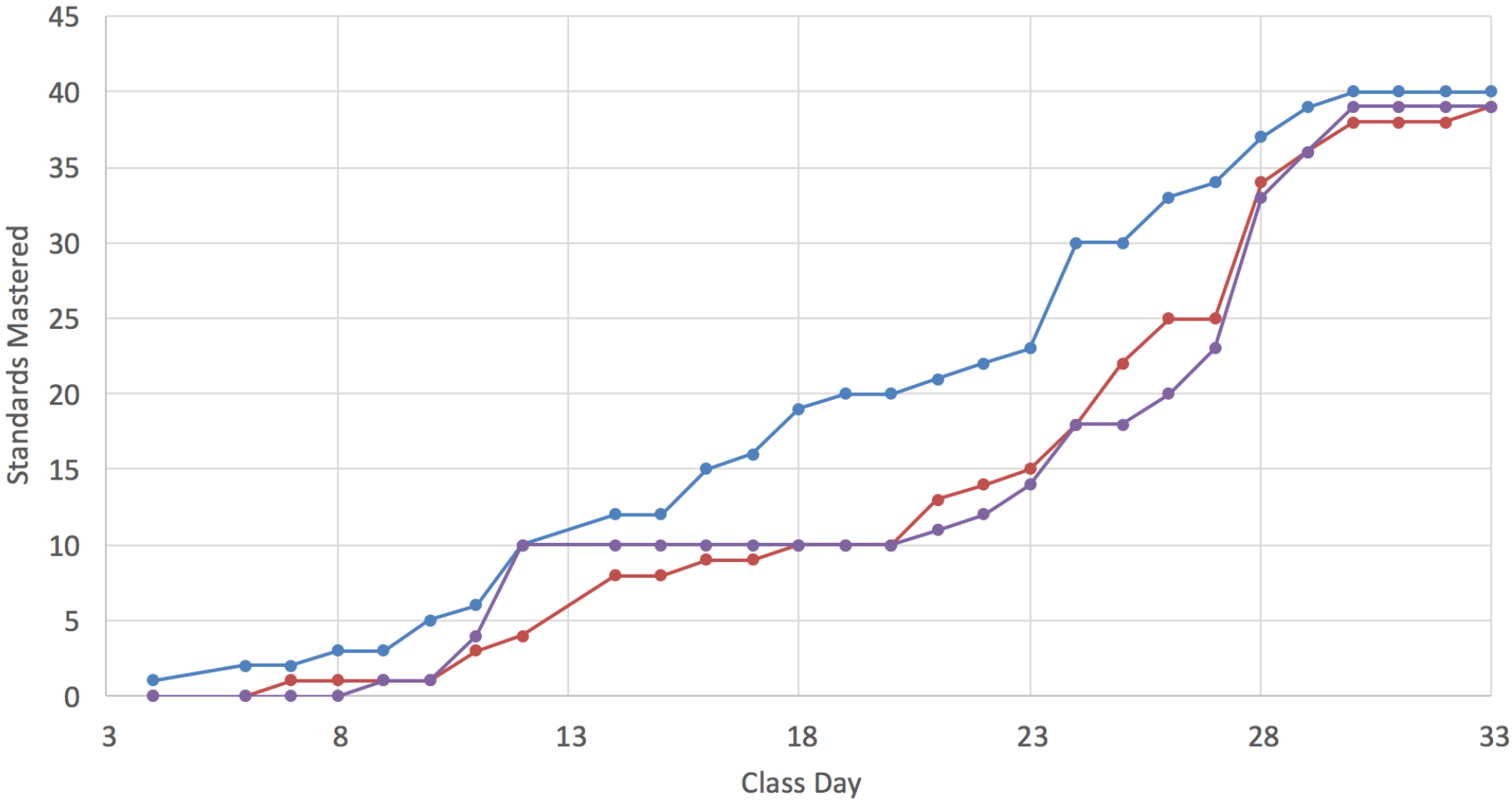
Y = Yoda (E), J = Jedi (M), P = Padowan (R), D = Droid (N)

Sheet for standards mastered

Standards	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Total C	Total S	Total C+S
Student A	2	2	2	1	2	2	1	2	2	2	1	1	2	1	2	2	1		1	1	1	1	1	1	1	1			1	1	28	10	38
Student B	1	2	2			2	1	1	2	2	2	1		1	1	1			1	1	1	1	1		1	1	1	1	1		19	10	29
Student C	1	2	1	1	1	1	1	1	2						1	1			1	1	1	1			1		1		1	1	13	8	21
Student D	2	2	2	2	2	1	1	1	1	2	1	1				1			1	1	1	1	1	1			1	1	1		19	9	28
Student E	2	2	2	2		2		2	2	2	1		1		2	2			1	1	1	1			1	1			1	1	22	8	30
Student F	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1		1	1	1	1	1	1	1	1	1		1	1	29	11	40

SBG data promotes evidence-based teaching

Growth Charts



Student Comments

1) Partial Credit Misconception:

"If I were to get a question wrong then I would not attempt it again **because there would be no point**. But with the mastery exams, I have the opportunity to go back, look at what I did wrong and fix my mistakes."

2) Increases Retention:

"I'm pretty much made to stay after and answer questions and I have seen mass improvement since day one! Since I have to keep re-addressing the problem it **makes me remember it better**!"

3) Increases Confidence:

"My confidence in solving problems has greatly improved, not only in math but **across all subjects**."