Implementing Standards Based Grading Across Lower Level Mathematics Courses

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Institutions and Courses

Courses:

- Precalculus
- Calculus 1, 2, 3
- Linear Algebra
- Differential Equations

Spring Hill College:

- Jesuit, Private, Liberal arts
- Around 1,200 students
- Class sizes: 25

Florida Southern College:

- Private, Liberal arts
- Around 2,400 students
- Class sizes: 13

University of Alabama:

- Public
- Around 38,000 students
- Class Sizes: 15, 75

University of South Alabama:

- Public
- Around 16,000 students
- Class Sizes: 9, 20, 38

Standards Based Grading

The course is split into 2 types of *standards*:

Core and Supplementary

- Precalculus: Solve an inequality using a sign chart. (Core)
- DE: Solve a problem using Newton's law of cooling. (Supp.)

Key features:

- Assessments are graded by standard
- Assessments use a binary rubric
- Students can initiate re-assessment opportunities
- Re-assessments can replace earlier scores

Marking Attempts

Each standard attempted is graded using an EMRN rubric:

- No Evidence: requires major revision/incomplete
- Re-assessment Needed: demonstrates some understanding but revisions are needed
- Meets Expectation: completely correct
- Exceeds Expectation (not always applicable)

In addition, we use a * mark for needs clarification/minor correction.

- alleviates frustration
- can be replaced with mastery

Grade Bundles in Differential Equations

Final Grade Checklist

Course grade	D	C	B	A
C standards mastered	☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐		☐ ☐ ☐ ☐
C standards continually mastered			☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐ ☐ ☐ ☐
S standards mastered		☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐ ☐	
Participation Score	☐50%	☐75%		
Homework Score	☐55%	☐75%	☐85%	☐95%

Advice on Regulating Reassessments

Ways to reassess:

- in-class written assessment (quiz or exam)

  try frequent quizzes, less exams

- office hours written assessment

  Be organized! (and students too)

- office hours oral assessment

  limit to one attempt per day / more conversational

- reassessment form

Keeping a Gradebook

Sheet for attempts

Assessment	Q1	OH	OH	OH	Q2		OH		OH	Q3	OH						Exam 1								OH						Q4
Day Count	4	6	7		8	8		9		10	10	11					12								14						15
Standard	S1	S2	S1	S2	S2	C1	C2	C3	S1	S2	C3	C4	C1	C2	C3	C5	S2	C1	C2	C3	C4	C5	C6	S2	S4	C1	C4	C5	C6	S2	S4	C7
Student A	J			J		J	J					*/J						J		J		P	J									Y
Student B	J				P	J	D					*/J		J	P		*/J	J	J	J	P	*/J	P	P								P
Student C	Y	J				J	P				J	J		Y				J	*/J	J	J	P	P					J	*/J			P
Student D	D		J		D	D	D					D	P	J	*/J		P	P	*/J	D	P	P	D	P		J	J	J	P	J		D
Student F	J					J	P					J		Y	*/J		Y	*/J	J	J	*/J	P	J		J							Y

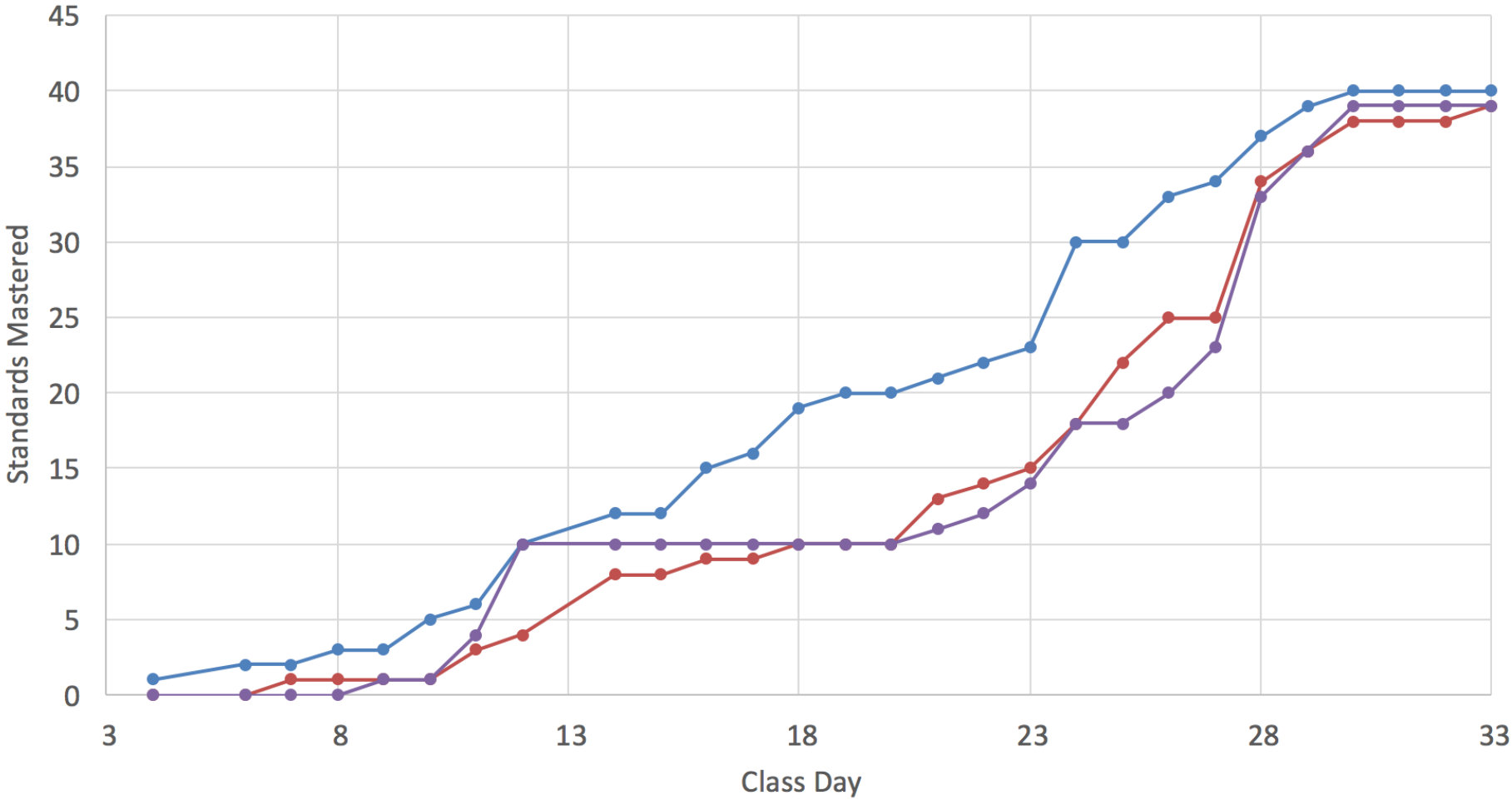
Y = Yoda (E), J = Jedi (M), P = Padowan (R), D = Droid (N)

Sheet for standards mastered

Standards	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	Total C	Total S	Total C+S
Student A	2	2	2	1	2	2	1	2	2	2	1	1	2	1	2	2	1		1	1	1	1	1	1	1	1			1	1	28	10	38
Student B	1	2	2			2	1	1	2	2	2	1		1	1	1			1	1	1	1	1		1	1	1	1	1		19	10	29
Student C	1	2	1	1	1	1	1	1	2						1	1			1	1	1	1			1		1		1	1	13	8	21
Student D	2	2	2	2	2	1	1	1	1	2	1	1				1			1	1	1	1	1	1			1	1	1		19	9	28
Student E	2	2	2	2		2		2	2	2	1		1		2	2			1	1	1	1			1	1			1	1	22	8	30
Student F	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1		1	1	1	1	1	1	1	1	1		1	1	29	11	40

SBG data promotes evidence-based teaching

Growth Charts



Beyond Anecdotes - Questions

1. We separatley have similar observations, how can we properly compare different courses across different institutions?
2. Are there any apparent effects on cheating?
3. Is there a way to quantify observations from a growth chart?
4. Which students tend to use office hours the most?

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Some Results from Precalculus

C2: solving inequalities

Attempt	Master	Journeyman	Apprentice
1	8	14	3
2	2	10	5
3	5	8	2
4	4	6	0

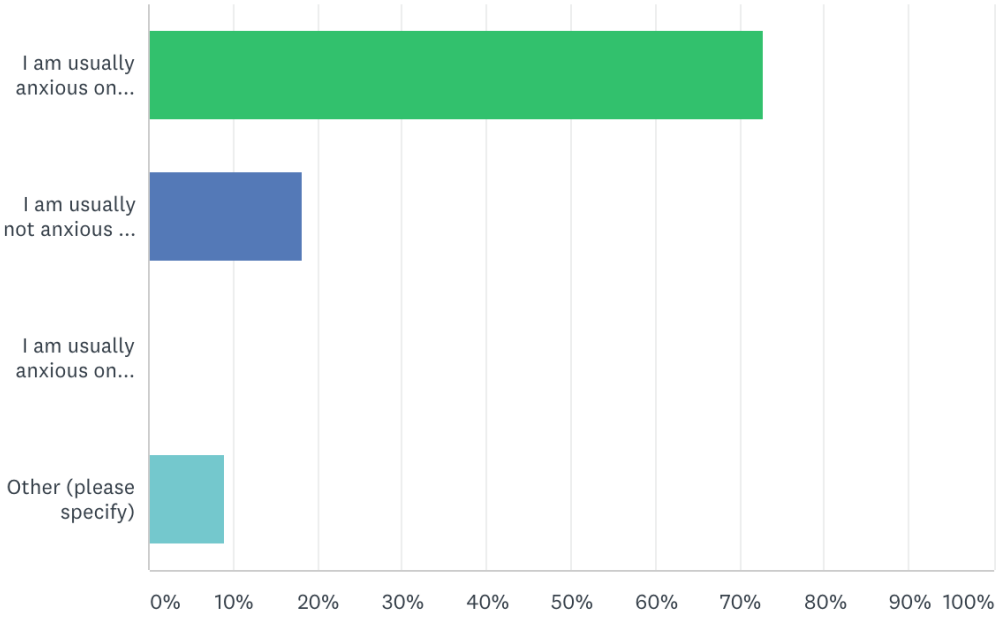
C13: finding vertex, max/min, inc/dec

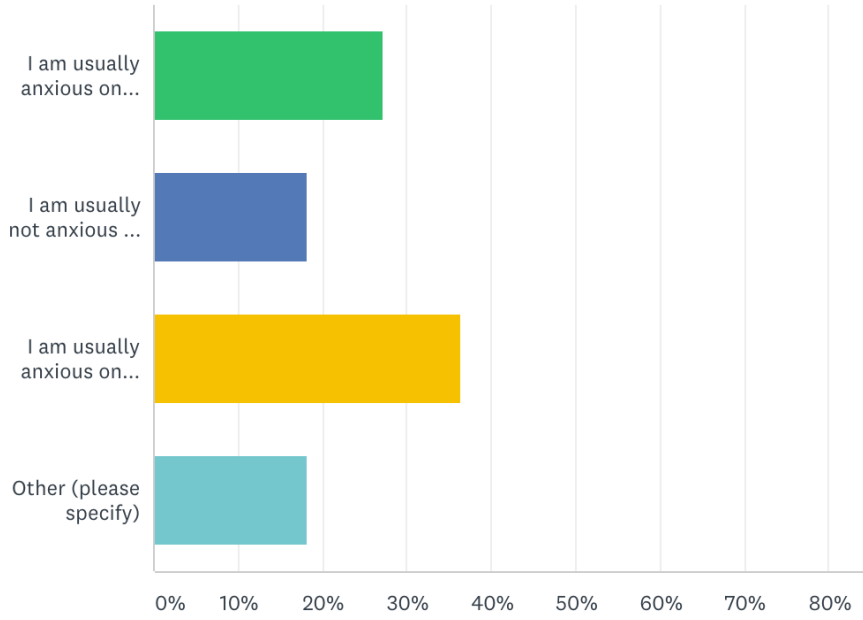
Attempt	Master	Journeyman	Apprentice
1	5	13	7
2	7	10	3
3	5	5	3
4	1	4	3

Advantages

Some advantages of mastery-based exams:

- less stressful
- several chances to display mastery
- see which parts of the course deserve more attention

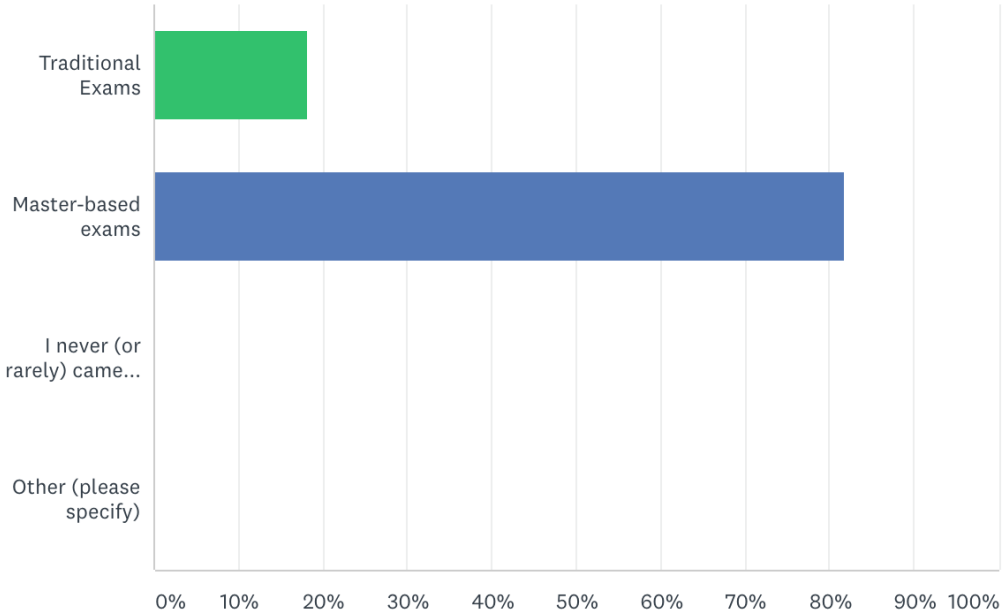


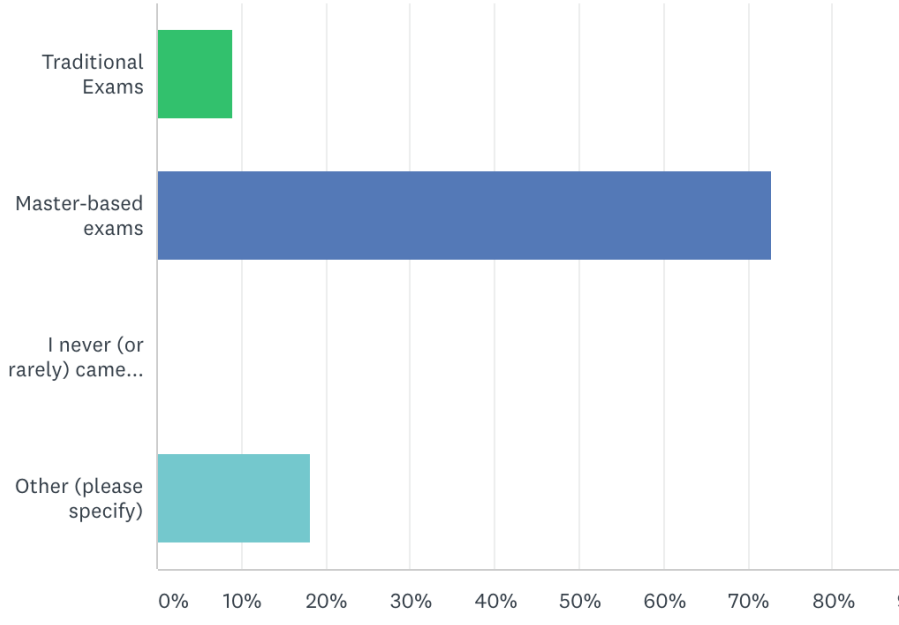


Advantages

Other advantages I observed:

- students learn from their mistakes
- became more aware of simple mistakes
- writing improved
- students came to office hours





An Example from Differential Equations

S2: Determine all IC's for which a given IVP has a unique solution.

Example: DE: $(y - x)y' = y + x$, IC: (a, b)

Step 1: First write $y' = \frac{y+x}{y-x}$

Step 2: The function $\frac{y+x}{y-x}$ is continuous when $y - x \neq 0$.

Step 3: The function $\frac{(1)(y-x)-(1)(y+x)}{(y-x)^2} = \frac{-2x}{(y-x)^2}$ is continuous when $y - x \neq 0$.

Step 4: The IVP has a unique solution for any IC (a, b) for which $a \neq b$.

Attempt 1

S2: For a given initial value problem, determine all initial conditions for which the system will have a unique solution.

1. Determine the largest region in the xy-plane for which the differential equation would have a unique solution whose graph passess through the point (a, b).

$$\begin{cases} (\sqrt{y} - 1)y' = x \\ y(a) = b \end{cases}$$

$y^{\frac{1}{2}} = \frac{1}{2} y^{\frac{-3}{2}}$

$f_y(x) = \sqrt{y} - 1 \, dy$

$= \frac{1}{2} y^{\frac{3}{2}}$

the function x is cont. everywhere

$f_y(x) = (\sqrt{y} - 1) y'$

D

Attempts 2 & 3

S2: For a given initial value problem, determine all initial conditions for which the system will have a unique solution.

1. Determine the largest region in the xy-plane for which the differential equation would have a unique solution whose graph passess through the point (a, b).

$$\begin{cases} (x^2+y^2)y' = y^2 \\ y(a) = b \end{cases}$$

$$y' = \frac{y^2}{x^2+y^2} \leftarrow \text{can't} = 0$$

$$f_y = \frac{2y(x^2+y^2) - y^2(0+2y)}{(x^2+y^2)^2}$$

$$f_y = \frac{2y(x^2+y^2) - y^2(2y)}{(x^2+y^2)^2}$$

$$f_y = \frac{2yx^2+2y^3-2y^3}{(x^2+y^2)^2}$$

$$f_y = \frac{2yx^2}{(x^2+y^2)^2} \leftarrow \text{can't} = 0$$

$(x^2+y^2)(x^2+y^2)$

$y^4+x^2y^2+x^2y^2+y^4$

~~x and y are continuous everywhere when $x^2+y^2>0$~~

this is continuous when $x^2+y^2>0$

S2: For a given initial value problem, determine all initial conditions for which the system will have a unique solution.

1. Determine the largest region in the xy-plane for which the differential equation would have a unique solution whose graph passess through the point (a, b).

$$\begin{cases} (y-x)y' = y+x \\ y(a) = b \end{cases}$$

$$(y-x)y' = y+x$$

$$y' = \frac{y+x}{y-x} \rightarrow \text{cannot equal zero}$$

$$f_y = \frac{(1+0)(y-x) - (y+x)(1-0)}{(y-x)^2}$$

$$f_y = \frac{(y-x)-(y+x)}{(y-x)^2}$$

$$f_y = \frac{y-x-y-x}{(y-x)^2}$$

$$f_y = \frac{-2x}{(y-x)^2} \rightarrow \text{cannot equal zero}$$

∴ this is contin. when $y-x \neq 0$.

Hence by the theorem of IVP this has a unique solution for when $b-a \neq 0$.

Student Comments

1) Partial Credit Misconception:

"If I were to get a question wrong then I would not attempt it again **because there would be no point**. But with the mastery exams, I have the opportunity to go back, look at what I did wrong and fix my mistakes."

Student Comments

2) Increases Retention:

"I'm pretty much made to stay after and answer questions and I have seen mass improvement since day one! Since I have to keep re-addressing the problem it **makes me remember it better**!"

Student Comments

3) Increases Confidence:

"My confidence in solving problems has greatly improved, not only in math but **across all subjects**."