Using Standards Based Grading in Mathematics Courses

Jason Elsinger, Ph.D.

Department of Mathematics
Florida Southern College

Conference on Teaching and Learning
USA, Mobile, AL
May 7th

The Colleges and Audience

Spring Hill College:

- Jesuit, Private
- Liberal arts
- Around 1200 students

Florida Southern College:

- Private
- Liberal arts
- Around 2400 students

Precalculus (lower level):

- Social science, mathematics and science majors
- Service course, typically difficult

Differential Equations (upper level):

- Mathematics, science majors
- Majors Course

The Standards

The course is split into 2 types of *standards*:

Core and Supplementary

Precalculus:

- C2: Solve an inequality using a sign chart.
- C7: Find the domain of a function.
- C13: Use the rate of change function to find the vertex of a parabola and where it increases and decreases.

Differential Equations

- C1: Given a physical situation, write a differential equation.
- S4: Solve a problem using Newton's law of cooling.
- S10: Write a system of differential equations in matrix form.

Mastery Based Assessment

Each standard attempted is given one of several marks:

- No Evidence: requires major revision/incomplete
- Re-assessment Needed: demonstrates some understanding but revisions are needed
- Meets Expectation: completely correct
- Exceeds Expectation

In addition, we use a * mark for needs clarification/minor correction.

- alleviates frustration
- can be replaced with mastery

Standards-Based Grading System

Key features:

- Assessments are graded by standard
- Standards are re-assessed throughout the course
- Course grades assigned by specs grading/primarily based on mastery
- Assessments use a binary rubric
- Students can initiate re-assessment opportunities
- Re-assessments can replace earlier scores

Grade Bundles in Differential Equations

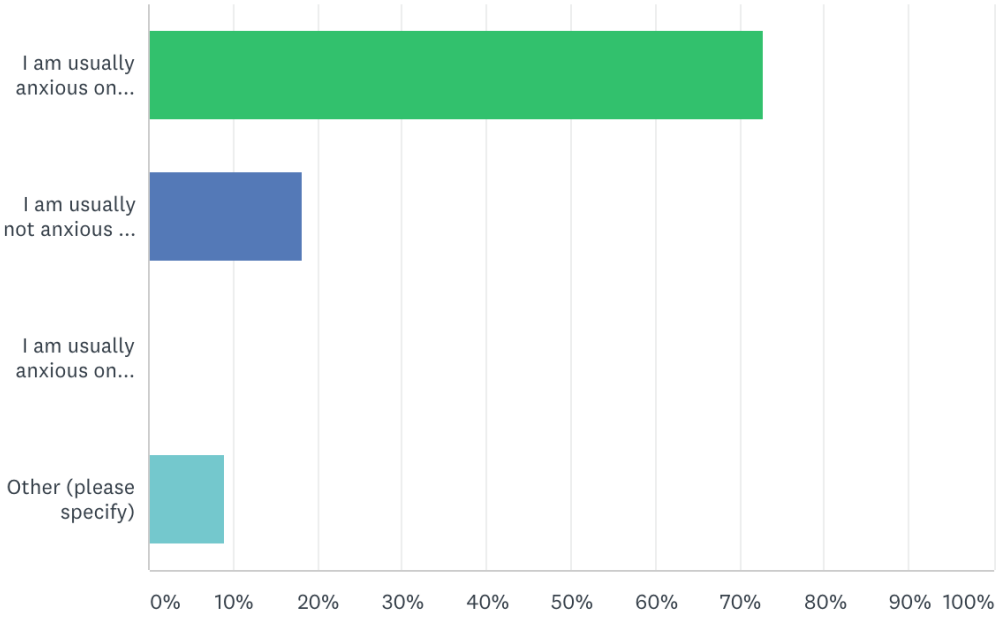
Final Grade Checklist

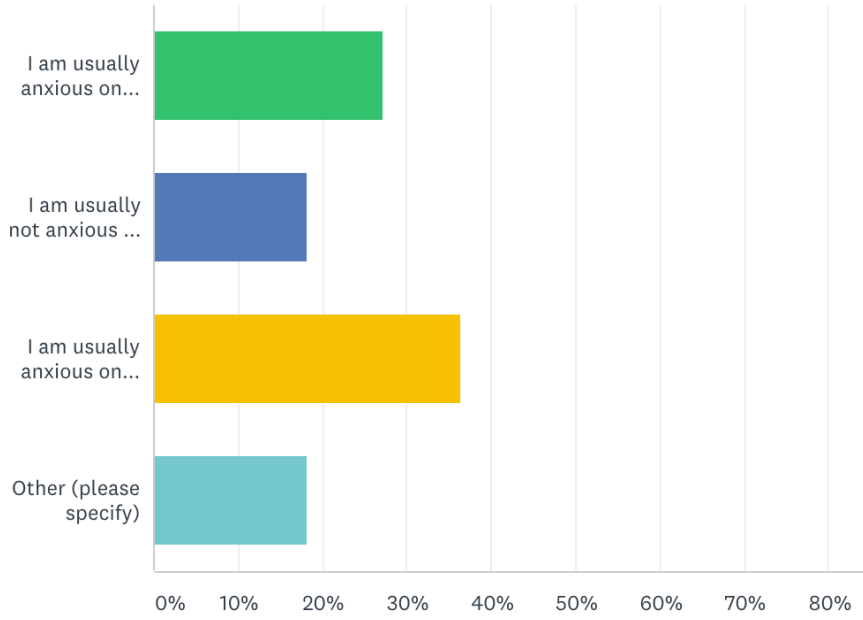
Course grade	D	C	B	A
C standards mastered	☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐		☐ ☐ ☐ ☐
C standards continually mastered			☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐ ☐ ☐ ☐
S standards mastered		☐ ☐ ☐ ☐ ☐ ☐	☐ ☐ ☐ ☐	
Participation Score	☐50%	☐75%		
Homework Score	☐55%	☐75%	☐85%	☐95%

Advantages

Some advantages of mastery-based exams:

- less stressful
- several chances to display mastery
- see which parts of the course deserve more attention

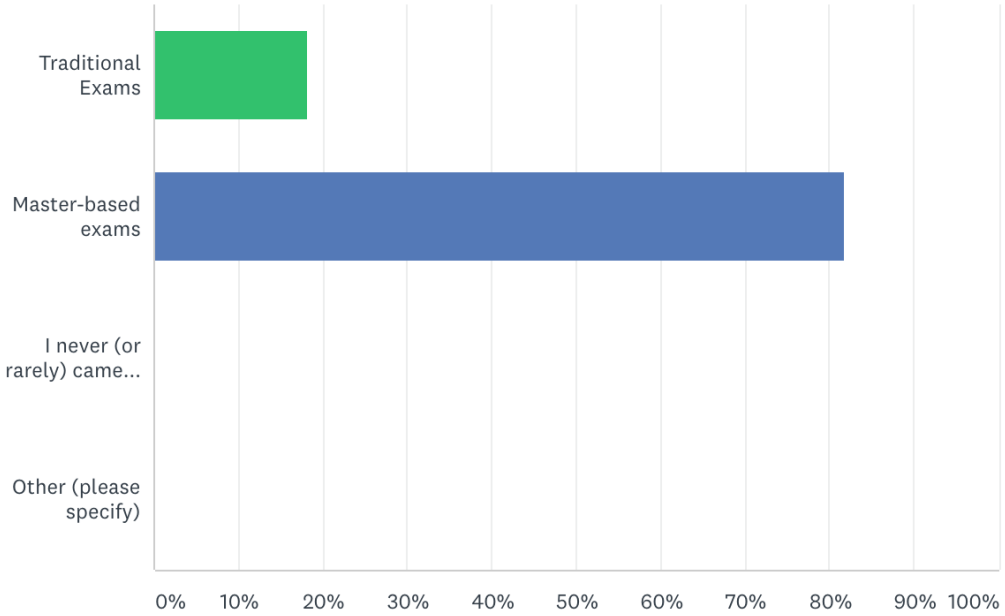


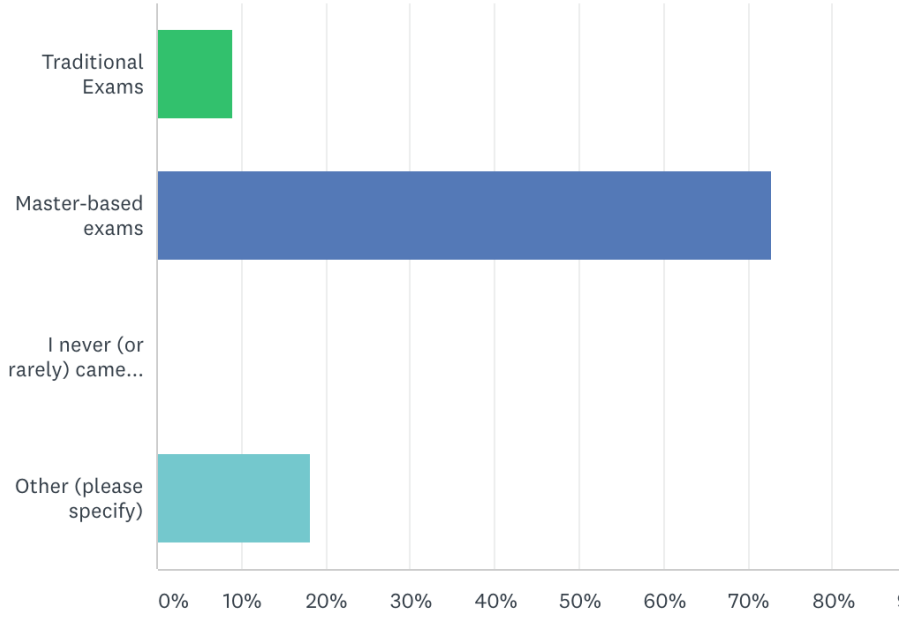


Advantages

Other advantages I observed:

- students learn from their mistakes
- became more aware of simple mistakes
- writing improved
- students came to office hours





An Example from Differential Equations

S2: Determine all IC's for which a given IVP has a unique solution.

Example: DE: $(y - x)y' = y + x$, IC: (a, b)

Step 1: First write $y' = \frac{y+x}{y-x}$

Step 2: The function $\frac{y+x}{y-x}$ is continuous when $y - x \neq 0$.

Step 3: The function $\frac{(1)(y-x)-(1)(y+x)}{(y-x)^2} = \frac{-2x}{(y-x)^2}$ is continuous when $y - x \neq 0$.

Step 4: The IVP has a unique solution for any IC (a, b) for which $a \neq b$.

Attempt 1

S2: For a given initial value problem, determine all initial conditions for which the system will have a unique solution.

1. Determine the largest region in the xy-plane for which the differential equation would have a unique solution whose graph passess through the point (a, b).

$$\begin{cases} (\sqrt{y} - 1)y' = x \\ y(a) = b \end{cases}$$

$y^{\frac{1}{2}} = \frac{1}{2}y^{\frac{-3}{2}}$

$f_y(x) = \sqrt{y} - 1 \, dy$

$= \frac{1}{2}y^{\frac{3}{2}}$

the function x is cont. everywhere

$f_y(x) = (\sqrt{y} - 1)\, y'$

D

Attempts 2 & 3

S2: For a given initial value problem, determine all initial conditions for which the system will have a unique solution.

1. Determine the largest region in the xy-plane for which the differential equation would have a unique solution whose graph passess through the point (a, b).

$$\begin{cases} (x^2+y^2)y' = y^2 \\ y(a) = b \end{cases}$$

$$y' = \frac{y^2}{x^2+y^2} \leftarrow \text{cant} = 0$$

$$f_y = \frac{2y(x^2+y^2) - y^2(0+2y)}{(x^2+y^2)^2}$$

$$f_y = \frac{2y(x^2+y^2) - y^2(2y)}{(x^2+y^2)^2}$$

$$f_y = \frac{2yx^2+2y^3-2y^3}{(x^2+y^2)^2}$$

$$f_y = \frac{2yx^2}{(x^2+y^2)^2} \leftarrow \text{cant} = 0$$

$(x^2+y^2)(x^2+y^2)$

$y^4+x^2y^2+x^2y^2+y^4$

~~x and y are continuous everywhere when $x^2+y^2>0$~~

this is continuous when $x^2+y^2>0$

S2: For a given initial value problem, determine all initial conditions for which the system will have a unique solution.

1. Determine the largest region in the xy-plane for which the differential equation would have a unique solution whose graph passess through the point (a, b).

$$\begin{cases} (y-x)y' = y+x \\ y(a) = b \end{cases}$$

$$(y-x)y' = y+x$$

$$y' = \frac{y+x}{y-x} \rightarrow \text{cannot equal zero}$$

$$f_y = \frac{(1+0)(y-x) - (y+x)(1-0)}{(y-x)^2}$$

$$f_y = \frac{(y-x)-(y+x)}{(y-x)^2}$$

$$f_y = \frac{y-x-y-x}{(y-x)^2}$$

$$f_y = \frac{-2x}{(y-x)^2} \rightarrow \text{cannot equal zero}$$

∴ this is contin. when $y-x \neq 0$.

Hence by the theorem of IVP this has a unique solution for when $b-a \neq 0$.

Keeping a Gradebook

Sheet for attempts

Assessment	Q1	OH	OH		OH	Q2		OH			OH	Q3	OH					Exam 1							OH						Q4
Day Count	4	6	7		8	8		9			10	10	11					12							14						15
Standard	S1	S2	S1	S2	S2	C1	C2	C3	S1	S2	C3	C4	C1	C2	C3	C5	S2	C1	C2	C3	C4	C5	C6	S2	S4	C1	C4	C5	C6	S2	S4	C7
Student A	J			J		J	J					*/J						J		J		P	J									Y
Student B	J				P	J	D					*/J		J	P		*/J	J	J	J	P	*/J	P	P								P
Student C	Y	J				J	P				J	J		Y				J	*/J	J	J	P	P					J	*/J			P
Student D	D		J		D	D	D					D	P	J	*/J		P	P	*/J	D	P	P	D	P		J	J	J	P	J		D
Student F	J					J	P					J		Y	*/J		Y	*/J	J	J	*/J	P	J		J							Y

Sheet for standards mastered

Standards	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12		Total C	Total S	Total C+S
Student A	2	2	2	1	2	2	1	2	2	2	1	1	2	1	2	2	1		1	1	1	1	1	1	1	1			1	1		28	10	38
Student B	1	2	2			2	1	1	2	2	2	1		1	1	1			1	1	1	1	1		1	1	1	1	1			19	10	29
Student C	1	2	1	1	1	1	1	1	2						1	1			1	1	1	1			1		1		1	1		13	8	21
Student D	2	2	2	2	2	1	1	1	1	2	1	1				1			1	1	1	1	1	1			1	1	1			19	9	28
Student E	2	2	2	2		2		2	2	2	1		1		2	2			1	1	1	1			1	1			1	1		22	8	30
Student F	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1		1	1	1	1	1	1	1	1	1		1	1		29	11	40

Some Results from Precalculus

C2: solving inequalities

Attempt	Master	Journeyman	Apprentice
1	8	14	3
2	2	10	5
3	5	8	2
4	4	6	0

C13: finding vertex, max/min, inc/dec

Attempt	Master	Journeyman	Apprentice
1	5	13	7
2	7	10	3
3	5	5	3
4	1	4	3

Student Comments

1) Partial Credit Misconception:

"If I were to get a question wrong then I would not attempt it again **because there would be no point**. But with the mastery exams, I have the opportunity to go back, look at what I did wrong and fix my mistakes."

Student Comments

2) Increases Retention:

"I'm pretty much made to stay after and answer questions and I have seen mass improvement since day one! Since I have to keep re-addressing the problem it **makes me remember it better**!"

Student Comments

3) Increases Confidence:

"My confidence in solving problems has greatly improved, not only in math but **across all subjects**."